Parent Virtual Workshop

Problem Solving in Mathematics

February 12, 2025

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Procedural vs. Conceptual

- Procedural: Knowing how to perform operations
  - Procedures are essential for efficiency and accuracy.
- Conceptual: Understanding why math works
  - Conceptual understanding allows students to tackle unfamiliar problems with confidence, explain their reasoning clearly, and apply math skills to real-world contexts.
- Both are essential for mathematical mastery
- Balance leads to deeper understanding

2 ~~3~~

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40	1200	280
5	150	35

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What is Flexibility with Numbers & Why It Matters

Flexibility in Mathematical Thinking is crucial because it allows students to approach problems from multiple perspectives and adapt their strategies based on the context of the problem. This adaptability fosters deeper understanding and promotes critical thinking skills, enabling students to:

1. **Solve Problems Efficiently**: Different problems may require different methods. Flexibility helps students choose the most effective approach.
2. **Understand Concepts**: When students can view a problem in various ways, they develop a stronger grasp of mathematical concepts, rather than just memorizing procedures.
3. **Encourage Creativity**: Flexible thinking encourages innovative solutions and the exploration of non-conventional methods, enhancing engagement and interest in mathematics.
4. **Build Resilience**: Students learn to persist through challenges, as they can pivot to alternative strategies when faced with difficulties.
5. **Connect Ideas**: Flexibility fosters connections between different mathematical concepts, aiding in the transfer of knowledge to new situations.

Encouraging Flexibility with Numbers

- Ask for explanations: Why do you think that works? Can you solve it a different way?
- Provide opportunities for flexible thinking: Use mental math in everyday activities.
- Examples:
  - K-2: Solve by making ten
  - 3-5: Multiply by breaking it into smaller steps
  - 6-8: Simplifying fractions by finding patterns or using equivalencies

Examples of Flexibility with Numbers

1. Multiple Strategies for Problem Solving

- When solving 27+48, a student might:
  - Use the traditional algorithm, stacking the numbers.
  - Break it down: 27+40=67+8=75
  - Round 48 to 50, add 27+50=77, and then subtract 2 to find 75.

2. Understanding Different Representations

- A student can represent the fraction ¾:
  - As a decimal: 0.75
  - As a percentage: 75%
  - Using visual models, such as a pie chart or number line.

Examples of Flexibility with Numbers

3. Applying Concepts Across Topics

- A student uses their understanding of multiplication to solve a division problem:
  - Knowing that 12÷4=3 can help them understand 4×3=12.

4. Revising Estimates and Calculations

- When estimating 59×6, a student may: Round 59 to 60 for a quick estimate of 360 (as a check). Calculate exactly to find 354 and compare with the estimate.

5. Using Patterns and Relationships

- **Example**: Recognizing that adding 9 is the same as adding 10 and then subtracting 1: For 47+9, a student might compute 47+10-1=56.

Examples of Flexibility with Numbers

6. **Exploring Algebraic Expressions**

- **Example**: When simplifying $2(x+3)+4$: A student can distribute to get $2x+6+4$ or recognize that they can combine like terms first to make the process easier.

7. **Using Graphs for Visualization**

- A student interpreting a line graph can switch between analyzing the slope and determining the y-intercept to understand the relationship between variables.

These examples illustrate how flexible thinking in mathematics enables students to approach problems creatively and effectively, fostering a deeper understanding of the subject.

25 x 14 =

Think about the power of tens and quarters:

25 x 10 = 250
25 x 4 = 100

So, 25 x 14 = 350

2,000
-1, 362

Think Scaling:

1,999
-1, 361
638

Problem Solving in Mathematics

What we will cover tonight:

Workshop Highlights:

- **Overview of Problem-Solving Strategies:** Explore a toolkit of strategies that can be applied to complex multi-step mathematics problems.
- **Practical Techniques:** Learn basic problem-solving methods that enhance understanding and problem-solving skills.
- **Supportive Role:** Gain insights into how to assist and guide children through their mathematics challenges.

Problem Solving in Mathematics

8 Standards of Mathematical Practice

Problem Solving Strategies

Supporting Math at Home

8 Standards of Mathematical Practice

1. Make Sense of Problems
2. Reason Abstractly
3. Construct Arguments
4. Model with Mathematics
5. Use Tools Strategically
6. Attend to Precision
7. Look for Structure
8. Look for Repeated Reasoning

Why Mathematical Practices Matter

- Develops critical thinking skills
- Builds problem-solving abilities
- Fosters persistence
- Creates lifelong mathematical thinkers
- Supports success in school and beyond

SMP1: Make Sense of Problems

- Understanding before solving
  - K-2: Using manipulatives for story problems
  - 3-5: Breaking down multi-step problems
  - 6-8: Working with algebraic equations
- Parent tip: Ask "What do you know about this problem? What do you think it is asking you?”

SMP 2: Reason Abstractly

- Connecting real-world to math concepts
  - K-2: Counting physical objects
  - 3-5: Converting measurements
  - 6-8: Interpreting graphs
- Parent tip: Use everyday examples

SMP 3: Construct Arguments

- Explaining mathematical thinking
  - K-2: Showing how to count and add
  - 3-5: Defending solution methods
  - 6-8: Proving mathematical concepts
- Parent tip: Ask "Why do you think that works? Can you explain your thinking?”

SMP 4: Model with Mathematics

- Representing problems visually
  - K-2: Drawing pictures for word problems
  - 3-5: Creating data tables
  - 6-8: Graphing relationships
- Parent tip: Encourage visual representations

SMP 5: Use Tools Strategically

- Selecting appropriate resources
  - K-2: Number lines and counters
  - 3-5: Rulers and calculators
  - 6-8: Graphing tools and spreadsheets
- Parent tip: Introduce relevant math tools

SMP 6: Attend to Precision

- Accuracy in calculation and communication
  - K-2: Writing numbers correctly
  - 3-5: Using proper units
  - 6-8: Clear mathematical explanations
- Parent tip: Practice checking and labeling work

SMP 7: Look for Structure

- Recognizing patterns and relationships
  - K-2: Understanding number relationships
  - 3-5: Multiplication patterns
  - 6-8: Algebraic structures
- Parent tip: Point out patterns in daily life

SMP 8: Look for Repeated Reasoning

- Finding shortcuts and patterns
  - K-2: Number patterns
  - 3-5: Multiplication by powers of 10
  - 6-8: Similar problem-solving processes
- Parent tip: Help identify mathematical shortcuts

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Problem Solving Strategies

Supporting Math at Home

Restate the problem in your own words

- Read the problem aloud.
- Restate the problem in your own words.
- What is it asking me?
- What do I know? What do I need to know?

Draw a Picture or Diagram

- Visualize the problem, then draw it
  - Problem: "A pizza is cut into 8 equal slices. If 3 people each eat 2 slices, how many slices are left?"
    - [Draw a circle divided into 8 pieces], Shade 6 pieces (2 slices × 3 people), Count remaining unshaded pieces, Answer: 8-(3×2)=2 slices left

Act It Out

- Use manipulatives to act out the problem (Snacks work great)
  - Problem: "Sarah started with 5 balloons. If she gives away 2 balloons to her friends, how many balloons does Sarah have left?"
    - Use cereal, buttons, pennies, etc. to act it out

Make a Table or Chart

- Organize data systematically
  - Problem: "List all possible combinations of coins that make 25 cents using quarters, dimes, and nickels."
    - Organized list:
      - 1 quarter
      - 2 dimes + 1 nickel
      - 1 dime + 3 nickels
      - 5 nickels

Guess and Check

- Start with an educated guess, test, and adjust

Look for Patterns

- Identify recurring elements or sequences
  - Problem: “Samantha is planting flowers in her garden. She has decided to plant the flowers in a specific pattern. On the first row, she plants 2 tulips. In the second row, she plants 4 tulips. In the third row, she plants 6 tulips. If this pattern continues, how many tulips will be planted in the fifth row? The tenth row?”
    - 2, 4, 6, 8, **10**, 12, 14, 16, 18, **20**

Simplify the Problem

- Break down complex problems into simpler parts
  - Problem: "A store is selling notebooks for $3 each. Sarah buys 4 notebooks and pays with a $20 bill. How much change will she receive?"
    - Break it down: Cost per notebook = $3, Number of notebooks = 4, Money given = $20
    - What do you need to know first?
    - Need to: multiply 3 × 4, then subtract from 20

Use Logical Reasoning

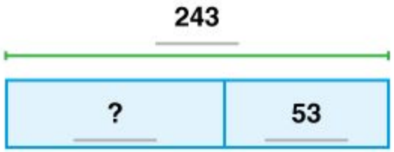
- Apply if-then thinking, eliminate impossible options

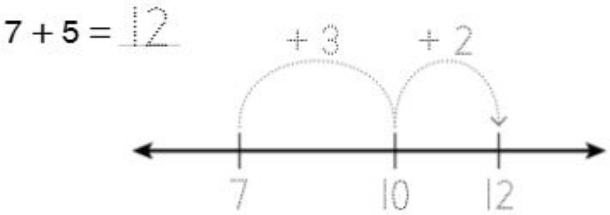
Working Backwards

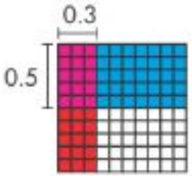
- Start with the answer and then work step by step in reverse
  - Problem: "After spending half her money on lunch and then $6 on a book, Jenny has $4 left. How much did she start with?"
    - Work backwards: Final amount = $4, Add book cost: 4+6=10, Double (since lunch was half): 10×2=20, Answer: $20 initially

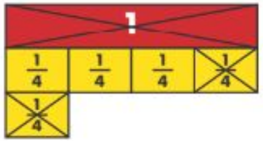
Create a Mathematical Model

- Represent the problem using mathematical symbols or equations
- Use a formula A = l x w









Strategies for Checking

- Is it reasonable?
- Label it
- Use the inverse operation
- Plug-in method (Great for algebra)

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Problem Solving Strategies

Supporting Math at Home

Supporting Math at Home

- Ask open-ended questions
- Encourage multiple approaches
- Encourage children to explain their thinking
- Celebrate effort and persistence
- Make math part of daily routines
- Create a positive math environment

Supporting Homework and Practice

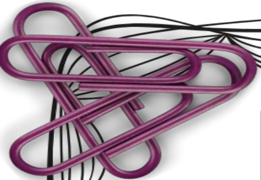
- Create a conducive study environment
- Offer guidance, not answers

Example: Guided Problem-Solving

- Read problem together
- Ask: "What information do we have? What are we solving for?"
- Parent Tip: "Could drawing a picture help?" (Your choice of strategy.)

Growth Mindset in Mathematics

- Embrace challenges
- Learn from mistakes
- Value effort over perfection
- Encourage persistence
- Celebrate progress



Real-World Applications

- Link math problems to everyday situations
- Cooking: Fractions and measurements
- Shopping: Budgeting and estimation
- Games: Math-based activities
- Sports: Statistics and scoring
- Travel: Distance, time, and cost

Q and A

Thank you for your attendance.

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